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# MODELING OF BOVINE SPONGIFORM ENCEPHALOPATHY IN A TWO-SPECIES FEEDBACK LOOP

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7 **Abstract**

Bovine spongiform encephalopathy, otherwise known as mad cow disease, can spread when an individual cow consumes feed containing the infected tissues of another individual, forming a one-species feedback loop. Such feedback is the primary means of transmission for BSE during epidemic conditions. Following outbreaks in the European Union and elsewhere, many governments enacted legislation designed to limit the spread of such diseases via elimination or reduction of one-species feedback loops in agricultural systems. However, two-species feedback loops—those in which infectious material from one-species is consumed by a secondary species whose tissue is then consumed by the first species—were not universally prohibited and have not been studied before. Here we present a basic ecological disease model which examines the rôle feedback loops may play in the spread of BSE and related diseases. Our model shows that there are critical thresholds between the infection's expansion and decrease related to the lifespan of the hosts, the growth rate of the prions, and the amount of prions circulating between hosts. The ecological disease dynamics can be intrinsically oscillatory, having outbreaks as well as refractory periods which can make it appear that the disease is under control while it is still increasing. We show that non-susceptible species that have been intentionally inserted into a feedback loop to stop the spread of disease do not, strictly by themselves, guarantee its control, though they may give that appearance by increasing the refractory period of an epidemic's oscillations. We suggest ways in which age-related dynamics and cross-species coupling should be considered in continuing evaluations aimed at maintaining a safe food supply.

8 *Keywords:* disease dynamics, mad cow disease, BSE, transmissible spongiform encephalopathy, prion,  
9 disease modeling

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10 **Introduction.** Bovine spongiform encephalopathy (BSE) is a disease in which a molecule of a specific  
11 protein misfolds into a pathogenic state; this misfolded protein then amplifies by inducing similar pathogenic  
12 misfoldings in other molecules of that protein. [Prusiner, 1997] For short-hand in this paper, we use the term  
13 “prion” to refer only to the misfolded form of the protein. The disease leads to neurodegeneration and death.  
14 It can be transmitted when a non-infected individual consumes prion-containing tissues from an infected  
15 individual. [Cummins et al., 2001; Wilesmith et al., 1992] In the late stages of the incubation period, the

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16 brain and spinal cord are known to have especially high levels of prions; prions are also known to be present  
17 in the peripheral nervous system and ileum, but to a lesser extent. [Arnold et al., 2009; Masujin et al., 2007;  
18 Wells et al., 1998]

19 Consider a hypothetical disease limited to vertebrates and transmitted when a susceptible individual  
20 consumes tissues from an infected one. The spread of the disease is normally self-limiting. Prey are consumed  
21 by predators, predators become prey, and the disease propagates along the food chain until non-susceptible  
22 invertebrate decomposers take charge.

23 Suppose, however, that the trophic structure is not a simple food chain, but rather a food web containing  
24 a feedback loop connecting two or more vertebrate species. For example, if a scavenger and predator species  
25 are trophically linked such that individual scavengers consume some dead predators, and living predators  
26 occasionally kill and consume the scavengers, then the dynamics are wholly different. The spread of the  
27 disease can progress cyclically around the feedback loop, limited not by the number of links in the chain but  
28 only by the size of the vertebrate populations.

29 Feedback loops can occur not just in nature, but also in agriculture. Livestock fed restricted diets often  
30 need food supplements, such as additional protein. Soybean meal can be used for this, but animal-derived  
31 protein is another source. Because large numbers of animals are slaughtered daily, and because not all of  
32 the slaughter is marketable to humans, a fraction remains. This fraction represents a prodigious quantity of  
33 material—up to 24 million tons or more per year in the U.S. alone [Kirstein, 1999]—that can be rendered  
34 into a diet supplement for livestock called meat-and-bone meal, among other names. Livestock may also  
35 be fed animal byproducts such as poultry litter. For example, in 2003, the state of Florida produced one  
36 million tons of poultry litter, 350,000 of which were available for use in feed. [Sapkota et al., 2007]

37 TSEs are known to spread among livestock such as cows [Prusiner, 1997; Nathanson et al., 1997; Wile-  
38 smith et al., 1988; Wells et al., 2007] and sheep [Detweiler and Baylis, 2003] when their feed is contaminated  
39 with infected tissues. To combat such spread, many countries have enacted legislation restricting what  
40 may be fed to susceptible species by either eliminating feedback loops altogether or prohibiting one-species  
41 feedback loops. Two-species loops are not, however, universally prohibited.

42 Our model considers a form of feedback in which prions are amplified in one species and then fed to a  
43 secondary species in which they may or may not be decreased before being fed back to the first species.  
44 Although the disease is also thought to propagate via direct maternal transmission [Donnelly et al., 1997;  
45 Wilesmith et al., 1997] and via cross-contamination of feed [Abrial et al., 2005; Wilesmith, 1996b,a], the  
46 former, if it occurs, does so only at levels insufficient to maintain an epidemic [Donnelly et al., 2002;  
47 Wilesmith et al., 2010] whereas the latter has been heavily regulated. Feedback through consumption of  
48 infected tissues is the primary means of BSE transmission during epidemic conditions [Cummins et al.,  
49 2001], so it seems worthwhile to consider this in the context of a two-species loop. This is especially so given  
50 that regulations prohibiting single-species cattle loops have created interest in making up the difference by

sourcing protein from other species. [Jenkins, 2006]

In the two-species feedback loops we consider, infectious material passes through a secondary species. In the worst-case scenario, the secondary species becomes infected and actively contributes to the growth of the disease, but it is not necessary that infection occur. There is also the possibility that the secondary species harbours infectious material for either long or short periods without ever developing symptoms.

Although it is not known whether the disease has transmitted in this way, it represents a possible means which has not been universally prohibited nor, to the best of our knowledge, considered by prior studies.

**Methods of analysis.** The aim of our model is to examine population dynamics and feedback loops under the most basic conditions, applying the simplest feasible model in order to expose underlying theoretical patterns and the relative importance of parameters in one- and two-species loops. Previous models have focused on different questions such as quantifying the real world risk of human and cattle exposure during and following the UK epidemic [Ferguson et al., 1999; Cohen et al., 2001, 2003; Cohen, 2006], the dynamics of the UK epidemic [Ferguson et al., 1997; Thornley and France, 2008], or the spread of BSE in the cells of a single individual [Nowak et al., 1998; Kellershohn and Laurent, 2001].

Instead of tracking the number of hosts that are infected, susceptible, and resistant, as is common in ecological disease models, this model simplifies that structure by tracking the total quantity of disease agents (prions in this case) resident within each host species, treating the hosts merely as an environment in which the disease exists.

Because the conditions which could lead to an epidemic are of primary interest, the model focuses on the early growth phase of the potential epidemic, when the disease would still be spreading undetected and mitigation measures would not be in effect. Individuals are not clearly symptomatic and, therefore, are neither being culled nor dying.

Although individual animals' susceptibility may vary, the model focuses on a subpopulation wherein all members are susceptible, and equally so. For simplicity, the model also assumes—as may be the case in an agricultural setting—that the lifespan of all hosts is artificially limited to a fixed number of years, that the size of the population is held constant, and that all individuals in an age-class are treated equally.

**One-species model.** The tightest possible feedback loop occurs when individual animals ingest tissues or byproducts of their own species, a practice which led to the spread of BSE in the United Kingdom and elsewhere. [Wilesmith et al., 1988, 1992] We show that for such a loop, there are critical combinations of lifespan, infectivity, and feedback below which the number of infections in a population will decrease with time and eventually vanish, and above which the infection will expand epidemically.

The model uses three parameters per species,  $x_i$ ,  $c_i$ , and  $R$ . The net total prion level of all animals in the herd in their  $i$ th year of life, at the beginning of that year, is represented by  $x_i$ . The prions are amplified

84 in infected tissue by a factor of  $R$  in each time unit. Upon slaughter, some fraction  $c_i$  of the tissues from  
 85 the oldest age class is fed back and incorporated into the tissues of the younger age class  $i$ . This term  
 86 also incorporates the probability of infection, which relates to the infectivity of the prions, dose size, and  
 87 heterogeneity of consumption.

88 In the model, the lifespan of the species is  $n$  years. In the United States and the United Kingdom, the  
 89 lifespan of most beef cattle is held close to 2–3 years, with breeding and dairy cattle living an average of  
 90 5–7 years, but with a minority of cattle living up to 16 years. [Donnelly et al., 2002; USDA, 2006]

91 The model could also be arranged to include the effects of slaughtering younger age-classes and feeding  
 92 them back. However, this would mean that fewer prions would make it to older age-classes leading to slower  
 93 growth of the disease. The more conservative formulation of the system we examine here assumes that all  
 94 members of a species make it to the same (old) age. This maximizes amplification resulting in stronger  
 95 conclusions concerning the efficacy of control measures.

96 For cattle, the consumption of even very small amounts of infectious tissue is sufficient to spread the  
 97 disease: Wells et al. [2007] experimentally determined that 50% of cattle would be clinically affected by  
 98 a dose of 0.20g, with no evidence for a minimum dose. Here we arbitrarily seed our model with a “prion  
 99 level” of 1. Despite the disease’s infectivity, clinical signs take an extended period to present themselves:  
 100 the incubation period in cattle is approximately 3–5 years. [Wells et al., 2007] Nowak et al. [1998] suggest  
 101 on theoretical grounds that within an individual the disease’s amplification is a trade-off between the linear  
 102 growth of prion aggregates and exponential growth caused by the fracturing of these aggregates, while  
 103 Arnold et al. [2009] experimentally determined that the disease had a doubling time of 1.2 months in the  
 104 central nervous system. This implies an exponential growth rate of  $R = 3$  per year, which we adopt here.

105 Given the foregoing, if an age-class is a year long, the number of prions in each age-class in the next time  
 106 unit is approximated by the following dynamical system.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & Rc_1 \\ R & 0 & \cdots & 0 & 0 & Rc_2 \\ 0 & R & \cdots & 0 & 0 & Rc_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & R & 0 & Rc_{n-1} \\ 0 & 0 & \cdots & 0 & R & Rc_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix}_t \quad (1)$$

107

108

The behavior of the system is governed by the eigenvalues of its characteristic polynomial

$$\lambda^n = c_n \lambda^{n-1} R + c_{n-1} \lambda^{n-2} R^2 + c_{n-2} \lambda^{n-3} R^3 + \cdots + c_2 \lambda R^{n-1} + c_1 R^n \quad (2)$$

109 In the discrete-time formulation of Equation 1, the quantity of prions,  $x_i$ , in each age-class will tend to  
 110 increase in the population if the absolute value of any eigenvalue  $\lambda$  of the matrix  $M_n$  is greater than 1. In  
 111 contrast, if the absolute values of all the eigenvalues are less than 1, the quantity of prions will tend towards

112 ZERO.

113 Therefore, the dividing line between expansion of the disease and its extinction occurs where  $\lambda = 1$ .  
 114 Under this condition, the characteristic polynomial takes the form

$$1 = R^{n+1} \sum_{i=1}^n c_i R^{-i} \quad (3)$$

115 [Figure 1 about here.]

116 In the simplest case, all the feedback fractions  $c_i$  are zero except for one,  $c_k$ . This represents a situation  
 117 in which animal protein supplements are given to each animal for one year only, year  $k$ . For instance, if  
 118  $k = 3$ , then animals are fed supplements only in their third year of life, beginning at age 2 and ending just  
 119 before age 3, as dairy cattle might be fed during their prime lactation period. In such a case, Equation 3  
 120 simplifies so that the threshold of age occurs where

$$n = k - \left( 1 + \frac{\ln c_k}{\ln R} \right) \quad (4)$$

121 If the lifespan,  $n$ , of the animals is short—less than the term to the right of the equal sign in Equation 4—the  
 122 disease dies out. If  $n$  is larger, the disease can spread. Figure 1 shows an example of disease dynamics on  
 123 both sides of the threshold, using the same initial conditions and parameters, but with differing lifespans.  
 124 However, Equation 3 is general and can be solved to find a critical threshold for any given set of parameters.

125 **Two-species feedback loop.** If the one-step feedback loop for a long-lived Species  $L$  is broken by  
 126 interposing a short-lived Species  $S$ —as in the cow–pig–cow loop permitted by current U.S. regulations—the  
 127 mathematics is similar to the one-species model above, but employs two embedded submatrices, one for  
 128 each species, coupled by the cross-feeding between the species. For example, if one species lives for three  
 129 years and the other for five, the matrix has the form

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}_{t+1} = \begin{pmatrix} 0 & 0 & 0 & - & - & - & - & R_L b_1 \\ R_S & 0 & 0 & - & - & - & - & R_L b_2 \\ 0 & R_S & 0 & - & - & - & - & R_L b_3 \\ - & - & R_S c_1 & 0 & 0 & 0 & 0 & 0 \\ - & - & R_S c_2 & R_L & 0 & 0 & 0 & 0 \\ - & - & R_S c_3 & 0 & R_L & 0 & 0 & 0 \\ - & - & R_S c_4 & 0 & 0 & R_L & 0 & 0 \\ - & - & R_S c_5 & 0 & 0 & 0 & R_L & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}_t \quad (5)$$

130  
 131 Here, dashes (-) are the same as zeros, placed for readability. Components  $x_{1,2,3}$  represent the prion  
 132 levels in the three age-classes of short-lived Species  $S$ , while components  $y_{1,2,\dots,5}$  represent the prion levels in  
 133 the five age-classes of longer-lived Species  $L$ .  $R_S$  and  $R_L$  generalize the amplification rate of the one-species

134 case, as described above. The portion of prions fed from the oldest age-class of one species to a particular  
 135 age-class  $i$  of the other is encapsulated in parameters  $b_i$  and  $c_i$ , along with the probability of infection.

136 Figure 2 depicts a numerical solution to Equation 5 in which prions amplify in the long-lived species  
 137 and decrease in the short-lived species. This demonstrates that a two-species loop does not in and of itself  
 138 prevent amplification nor eliminate the possibility of spread.

139 [Figure 2 about here.]

140 More generally, it can be shown that the characteristic polynomial of the generalized two-species system  
 141 with eigenvalue  $\lambda = 1$  takes the form

$$1 = R_S^{m+1} R_L^{n+1} \sum_{j=1}^m b_j R_S^{-j} \sum_{i=1}^n c_i R_L^{-i} \quad (6)$$

142 where  $m$  is the lifespan of the short-lived species,  $R_S$  is the amplification factor of that species,  $b_i$  represents  
 143 the infectivity of the prions and the dose being fed back from a susceptible long-lived species, and the other  
 144 variables are as before. From this it follows that the two-species system has a critical threshold for every  
 145 combination of amplification, feedback, and lifespan.

146 **Discussion.** In response to the proven risk of the single-species loop, the European Union prohibits the  
 147 incorporation of animal protein in any farmed livestock feed. The United States bans mammalian protein  
 148 in ruminant feed, excluding (a) blood and blood products, (b) inspected meats used for human food and  
 149 then heat processed for feed, and (c) meat consisting entirely of swine, horse, or poultry protein. [Dyckman  
 150 et al., 2002] Both regulatory structures have provisions intended to prevent cross-contamination of feed and  
 151 require separation and safe disposal of certain high-risk materials known to concentrate BSE, such as brains,  
 152 eyes, spinal columns, and distal ilea.<sup>1</sup>

153 The United States regulatory structure does not prohibit a two-species loop in which ruminant protein is  
 154 fed to pigs, horses, or poultry and their protein subsequently back to ruminants. Above, it was shown that  
 155 the two-species model has critical thresholds beyond which the disease may expand in both the short- and  
 156 long-lived species, with the longer-lived species establishing and maintaining the infection. The risk of this  
 157 occurring in our model may be assessed by considering the sensitivity of the disease's spread to variation in  
 158 its amplification rate, the amount of infectious material fed back through a loop, when this feedback occurs,  
 159 and the induced lifespans of the species involved.

160 In the one-species loop, for small values of  $c$ , the threshold of Equation 3 is approximated by considering  
 161 only the terms corresponding to the lowest age-class with a non-zero  $c$  value; this results in Equation 4.

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<sup>1</sup>See 21CFR589.2000, 21CFR589.2001, and Regulation (EC) No 999/2001

162 Figure 3a shows the sensitivity of the threshold to variation in the first age-class’ feedback amount  $c_1$  and  
 163 implies that even a feedback value as low as 1% is sufficient to drive epidemic expansion for moderate  
 164 amplification values in populations maintained at 4–5 years ( $R = \{5, 3\}$ , respectively). In contrast, for  
 165 populations maintained at two years—as is the case with much of the U.S. herd [USDA, 2006]—effective  
 166 separation can prevent epidemic growth for a range of amplification values in this model, provided materials  
 167 from older age-classes can reliably be kept separate.

168 Figure 3b shows the sensitivity of the threshold to variation in the age at which supplemental feeding  
 169 begins, or, along the other axis, to variation in the induced lifespan. Increasing the age of initial feeding  
 170 or decreasing the induced lifespan prove to be the most effective ways to reduce the growth of the disease,  
 171 yielding a linear response across all values. In contrast, varying the feedback amount  $c$  requires exponentially  
 172 more effort to achieve increasing levels of safety.

173 [Figure 3 about here.]

174 In a two-species feedback loop, the threshold is given by solving Equation 6. If the feedback values are  
 175 relatively small, then the system’s behavior is dominated by the youngest age-class of the susceptible species  
 176 and the oldest age-class of the non-susceptible species to receive infectious material. This configuration  
 177 is also the minimal level of intercession for a secondary species in this model and, as such, is the worst  
 178 case; other configurations will result in slower growth of the disease. In this scenario, the threshold is  
 179 approximated by

$$n \approx \frac{-\ln R_S - \ln(b_m c_1)}{\ln R_L} \quad (7)$$

180 Figure 3c shows the sensitivity of the two-species loop to variation in  $c_1$ , the feedback from the non-  
 181 susceptible to the susceptible species. Feedback from the susceptible to the non-susceptible species is assumed  
 182 to be small due to mandated separation of specified high-risk materials, and further reduced by an “inter-  
 183 species barrier” to transmission. [Wells et al., 2003] Once in the secondary species, the scenario depicted in  
 184 this figure assumes that the prion level remains constant ( $R_S = 1$ ) and that the prions are introduced only  
 185 in the non-susceptible species’ final year of life. This limiting case favors the growth of the disease, yet even  
 186 so, for low feedback values the age threshold is elevated to twice that of the single-species loop, or more.  
 187 In summary, effective separation in a single-species loop is useful in reducing the possibility of growth, but  
 188 does not eliminate it; thorough separation prior to feedback between two species is much more effective.

189 The distribution of ages in the U.S. herd is markedly weighted towards younger animals, with sharp drop-  
 190 offs in population levels following both the first and second year in both the beef and dairy herds. [USDA,  
 191 2006] If all animals fed supplements draw from a common pool, this implies that the majority of infectious  
 192 material ends up in short loops where the infection is not sustainable, provided there is efficient separation  
 193 of infectious materials from carcasses. This would reduce the possibility of epidemic expansion in both the  
 194 one- and two-species loops.

195 Similarly, the dose size any given animal receives is related to the homogeneity with which infectious  
196 material is mixed with non-infectious material in the feed production process. Higher degrees of homogeneity  
197 will correspond to smaller values of  $c$  and  $b$ .

198 It is important to note that the dynamics of the two-species loop do not depend on the secondary species  
199 actually contracting the disease, a possibility that is still hypothetical in the cow–pig–cow loop. The sole  
200 demand is that the disease be resident long enough for the secondary species to be fed back to the first. It  
201 is possible that a secondary species could consume and passively carry infectious material for long periods  
202 without ever metabolizing it or developing visible symptoms ( $R_S = 1$ ), as can be the case with heavy metals.  
203 The infectious material may also degrade in the secondary species ( $R_S < 1$ ), or be present only present for  
204 the amount of time it takes it to pass through the secondary species' gastrointestinal tract ( $R_S \approx 1, b \approx 0$ ).

205 Both Figure 1 and 2 show cyclic fluctuations in prion levels over time. This is not an artifact of the  
206 model, but inherent to the nature of the disease and the feedbacks. In an SIR model, infected individuals  
207 may coexist with the susceptibles they infect, but this is not the case here: the infection of new individuals  
208 must coincide with the death of the individual that infects them. If this death is accompanied by a reduction  
209 in the net prion level and feedback is restricted to a subset of the age-classes, a cyclic pattern emerges in  
210 the early stages of the epidemic. In the case of the single-species loop, such cycles have a period equal to  
211 the induced lifespan of the species. In the case of the two-species loop, the cycles have a period equal to the  
212 combined lifespans of the two species.

213 In the more general scenario of contaminated materials being fed back to multiple age-classes, cycles are  
214 still present but become increasingly dispersed over time, as shown in Figure 4. The more age-classes which  
215 are simultaneously exposed to the infection, the greater this dispersion is. Still, in a relatively-regimented  
216 situation such as is depicted in the figure, the behavior of the system is dominated by the youngest age-class  
217 of the susceptible species and the oldest age-class of the non-susceptible species to be fed contaminated  
218 material and the peaks of the cycles remain roughly the same initially, as Equation 6 suggests.

219 A lesson to be drawn from the single-species example is that, if a species never develops a transmissible  
220 spongiform encephalopathy or related disease, it may be that (1) the species is not susceptible to such  
221 diseases—that is, prions cannot be amplified within its tissues and disease symptoms do not manifest  
222 themselves; (2) the species does not interact with prions, though the possibility of the prions being absorbed  
223 and passively carried remains; (3) that the species is indeed susceptible, but that the induced lifespan is  
224 below the threshold, so the disease will not spread widely and will never be manifest in detectable quantities;  
225 or, (4) that the amount of material being fed between age-classes is so small as to prevent the disease from  
226 growing.

227 The emergence of BSE among cows may have started from a rare event, but ultimately it was the  
228 dynamics of the feeding system which allowed the disease to spread. Similar diseases may exist for other  
229 species but, for the above reasons, have not yet and may never emerge. Similarly, if a two-species system



230 never exhibits such diseases it is not necessarily because the system is protective in an absolute sense, but  
231 that it is not conducive to spread and amplification.

232 [Figure 4 about here.]

233 **Conclusions.** The thoughts presented here show that it is mathematically possible for cross-species  
234 feedback to spread BSE and related diseases through a susceptible population of animals, even though a  
235 species that does not exhibit susceptibility is interposed to break the feedback loop. In the process, the non-  
236 susceptible animals could actually be contaminated with the disease, albeit at lower levels. The consequences  
237 are more than an economic issue, for humans are believed to be susceptible to the disease and may contract  
238 it by ingesting infected meat. The result is called (new) variant Creutzfeldt–Jakob disease (nvCJD). [Hill  
239 et al., 1997; Scott et al., 1999; Prusiner, 1997]

240 In the case of agricultural livestock, the maximal solution is to eliminate all use of animal byproducts  
241 in livestock feed, similar to European regulations. Short of that, the minimal safe solution is to impose  
242 lifespan limits or reduce feedback below critical thresholds, thereby driving the disease to extinction even if  
243 it occasionally gets reintroduced.

244 The following steps are indicated by this study as possible ways to help push the system below these  
245 thresholds: (1) That animals symptomatic for the disease be prohibited in animal food supplements, though  
246 this really goes without saying. (2) That older animals, which will have experienced the most amplification,  
247 be eliminated from animal food supplements. (3) That animals with longer lifespans not be fed animal  
248 food supplements, especially in younger years. (4) That effective separation procedures be utilized when  
249 materials are fed both within and between species. (5) That quantities of animal food supplements be  
250 reduced. (6) That artificial limits to animal life-span be imposed. (7) That multi-step feedback loops be  
251 eliminated where dangers exist.

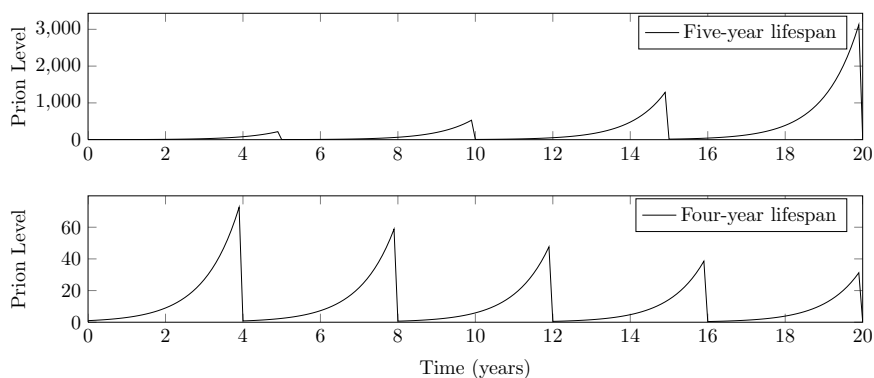
252 Insofar as separation procedures are effective, our model shows that two-species loops reduce risk but  
253 cannot be absolutely guaranteed to eliminate it, though adverse effects in our model generally arise only  
254 for what seem to be large parameter values. However, until these systems are well-understood, prudence  
255 requires that potential feedback loops be sought out and closely scrutinized. Lessons should be drawn from  
256 all sources, including these and other models of disease dynamics, for guidance in the on-going evaluation  
257 of regulations surrounding this class of diseases.

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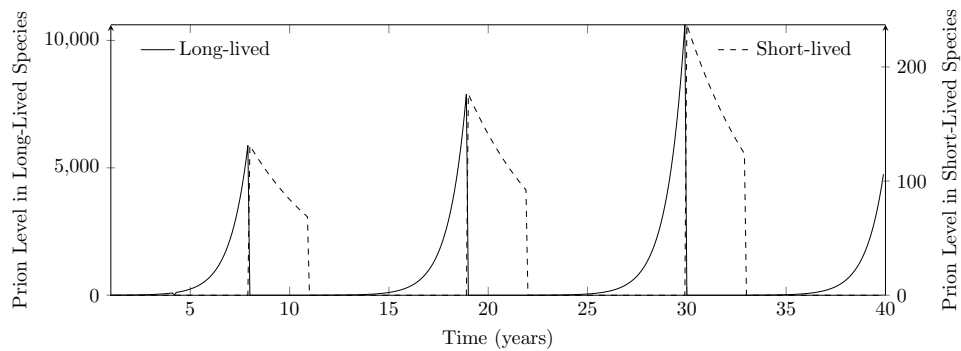
263 **Contributions.** Both authors contributed equally. CL conceived the modeling approach for one- and  
264 two-species loops, performed the initial simulations, and wrote the initial draft. RB further developed the  
265 mathematical analysis, incorporated a literature review, and revised the manuscript. Both authors jointly  
266 contributed to the final presentation.

- 267 Abrial, D., Calavas, D., Jarrige, N., Ducrot, C., 2005. Poultry, pig and the risk of BSE following the feed ban in france — a  
268 spatial analysis. *Veterinary Research* 36, 615–628. doi: <http://dx.doi.org/10.1051/vetres:2005020>
- 269 Arnold, M., Hawkins, S., Green, R., Dexter, I., Wells, G., 2009. Pathogenesis of experimental bovine spongiform encephalopathy  
270 (BSE): estimation of tissue infectivity according to incubation period. *Veterinary Research* 40 (1), 8–8. doi: <http://dx.doi.org/10.1051/vetres:2008046>
- 271
- 272 Cohen, J., 2006. Harvard model of bovine spongiform encephalopathy implications of importing cattle over 30 months of age  
273 from canada.
- 274 Cohen, J., Duggar, K., Gray, G., Kreindel, S., Gubara, H., HabteMariam, T., Oryang, D., Tameru, B., 2003. Prions and  
275 mad cow disease. CRC Press, Ch. 3: A Simulation Model for Evaluating the Potential for Spread of Bovine Spongiform  
276 Encephalopathy in Animals or to People, pp. 61–134.
- 277 Cohen, J., for Risk Analysis, H. C., for Computational Epidemiology, T. U. C., 2001. Evaluation of the potential for bovine  
278 spongiform encephalopathy in the United States. Harvard Center for Risk Analysis, Harvard School of Public Health.
- 279 Cummins, E., Grace, P., McDonnell, K., Ward, S., Fry, D., 2001. Predictive modelling and risk assessment of BSE: a review.  
280 *Journal of Risk Research* 4 (3), 251–274. doi: <http://dx.doi.org/10.1080/13669870152023809>
- 281 Detweiler, L., Baylis, M., 2003. The epidemiology of scrapie. *Revue Scientifique et Technique de l’OIE* 22 (1), 121–143.
- 282 Donnelly, C., Ferguson, N., Wilesmith, J., Anderson, R., 1997. Analysis of dam–calf pairs of bse cases: confirmation of a  
283 maternal risk enhancement. *Proceedings of the Royal Society of London. Series B: Biological Sciences* 264 (1388), 1647–  
284 1656. doi: <http://dx.doi.org/10.1098/rspb.1997.0229>
- 285 Donnelly, C. A., Ferguson, N. M., Ghani, A. C., Anderson, R. M., 2002. Implications of bse infection screening data for the  
286 scale of the british bse epidemic and current european infection levels. *Proceedings of the Royal Society of London. Series*  
287 *B: Biological Sciences* 269 (1506), 2179–2190. URL [http://rspb.royalsocietypublishing.org/content/269/1506/2179.](http://rspb.royalsocietypublishing.org/content/269/1506/2179.abstract)  
288 [abstract doi: http://dx.doi.org/10.1098/rspb.2002.2156](http://dx.doi.org/10.1098/rspb.2002.2156)
- 289 Dyckman, L. J., Lansburgh, E., Williams, C., Dishmon, J., Ryba, S., Turner, J., Holliday, J., Johnson, B., El-Osta, B., Shulman,  
290 C. H., 1 2002. Mad cow disease: Improvements in the animal feed ban and other regulatory areas would strengthen U.S.  
291 prevention efforts. Tech. rep., United States General Accounting Office.
- 292 Ferguson, N., Donnelly, C., Woolhouse, M., Anderson, R., 1997. The epidemiology of BSE in cattle herds in Great Britain.  
293 II. model construction and analysis of transmission dynamics. *Philosophical Transactions of the Royal Society of London.*  
294 *Series B: Biological Sciences* 352 (1355), 803. doi: <http://dx.doi.org/10.1098/rstb.1997.0063>
- 295 Ferguson, N., Donnelly, C., Woolhouse, M., Anderson, R., 1999. Estimation of the basic reproduction number of BSE: the  
296 intensity of transmission in British cattle. *Proceedings of the Royal Society of London. Series B: Biological Sciences* 266 (1414),  
297 23–32. doi: <http://dx.doi.org/10.1098/rspb.1999.0599>
- 298 Hill, A. F., Desbruslais, M., Joiner, S., Sidle, K. C. L., Gowland, I., Collinge, J., 10 1997. The same prion strain causes vCJD  
299 and BSE. *Nature* 389, 448–450. doi: <http://dx.doi.org/10.1038/38925>
- 300 Jenkins, T. C., 1 2006. Essential Rendering: All about the Animal By-Products Industry. National Renderers Association, Ch.  
301 Rendered Products in Ruminant Nutrition, pp. 111–124.
- 302 Kellershohn, N., Laurent, M., 2001. Prion diseases: dynamics of the infection and properties of the bistable transition. *Bio-*  
303 *physical Journal* 81 (5), 2517–2529. doi: [http://dx.doi.org/10.1016/S0006-3495\(01\)75897-3](http://dx.doi.org/10.1016/S0006-3495(01)75897-3)
- 304 Kirstein, D. D., 4 1999. Composition and quality of porcine meat and bone meal. *Tri-State Dairy Nutrition Conference, Citeseer,*  
305 p. 222.
- 306 Masujin, K., Matthews, D., Wells, G., Mohri, S., Yokoyama, T., 2007. Prions in the peripheral nerves of bovine spongiform  
307 encephalopathy-affected cattle. *Journal of general virology* 88 (6), 1850–1858.
- 308 Nathanson, N., Wilesmith, J., Griot, C., 6 1997. Bovine spongiform encephalopathy (BSE): Causes and consequences of a  
309 common source epidemic. *American Journal of Epidemiology* 145 (11), 959–969.
- 310 Nowak, M., Krakauer, D., Klug, A., May, R., 1998. Prion infection dynamics. *Integrative Biology Issues News and Reviews*  
311 1 (1), 3–15. doi: [http://dx.doi.org/10.1002/\(SICI\)1520-6602\(1998\)1:1<3::AID-INBI2>3.0.CO;2-9](http://dx.doi.org/10.1002/(SICI)1520-6602(1998)1:1<3::AID-INBI2>3.0.CO;2-9)
- 312 Prusiner, S., 10 1997. Prion diseases and the BSE crisis. *Science* 289, 245–251. doi: [http://dx.doi.org/10.1126/science.278.](http://dx.doi.org/10.1126/science.278.5336.245)  
313 [5336.245](http://dx.doi.org/10.1126/science.278.5336.245)
- 314 Sapkota, A., Lefferts, L., McKenzie, S., Walker, P., 2007. What do we feed to food-production animals? a review of animal  
315 feed ingredients and their potential impacts on human health. *Environmental health perspectives* 115 (5), 663. doi: <http://dx.doi.org/10.1289/ehp.9760>
- 316
- 317 Scott, M., Will, R., Ironside, J., Nguyen, H., Tremblay, P., DeArmond, S., Prusiner, S., 1999. Compelling transgenetic evidence  
318 for transmission of bovine spongiform encephalopathy prions to humans. *Proceedings of the National Academy of Sciences*  
319 *of the United States of America* 96 (26), 15137. doi: <http://dx.doi.org/10.1073/pnas.96.26.15137>
- 320 Thornley, J., France, J., 2008. Modelling bovine spongiform encephalopathy. *The Journal of Agricultural Science* 146 (02),  
321 183–194. doi: <http://dx.doi.org/10.1017/S0021859607007629>

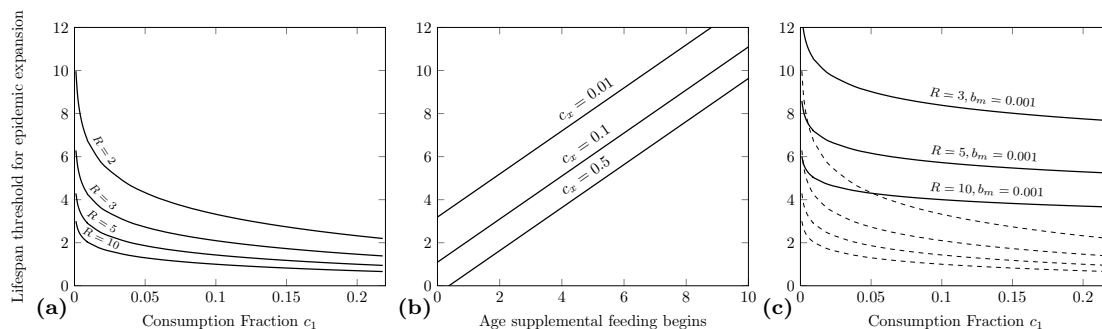
- 322 USDA, 7 2006. An estimate of the prevalence of BSE in the United States. Tech. rep., USDA Animal and Plant Health  
323 Inspection Service.
- 324 Wells, G., Konold, T., Arnold, M., Austin, A., Hawkins, S., Stack, M., Simmons, M., Lee, Y., Gavier-Widén, D., Dawson, M.,  
325 et al., 2007. Bovine spongiform encephalopathy: the effect of oral exposure dose on attack rate and incubation period in  
326 cattle. *Journal of General Virology* 88 (4), 1363–1373. doi: <http://dx.doi.org/10.1099/vir.0.82421-0>
- 327 Wells, G. A. H., Hawkins, S. A. C., Austin, A. R., Ryder, S. J., Done, S. H., Green, R. B., Dexter, I., Dawson, M., Kimberlin,  
328 R. H., 2003. Studies of the transmissibility of the agent of bovine spongiform encephalopathy to pigs. *Journal of General*  
329 *Virology* 84 (4), 1021–1031. doi: <http://dx.doi.org/10.1099/vir.0.18788-0>
- 330 Wells, G. A. H., Hawkins, S. A. C., Green, R. B., Austin, A. R., Dexter, I., Spencer, Y. I., Chaplin, M. J., Stack, M. J.,  
331 Dawson, M., 1998. Preliminary observations on the pathogenesis of experimental bovine spongiform encephalopathy (bse):  
332 an update. *Veterinary Record* 142 (5), 103–106. doi: <http://dx.doi.org/10.1136/vr.142.5.103>
- 333 Wilesmith, J., 1996a. *Bovine Spongiform Encephalopathy: The BSE Dilemma*. Springer, p. 45.
- 334 Wilesmith, J., 1996b. *Prion Diseases (Methods in Molecular Medicine)*. Humana Press, p. 155.
- 335 Wilesmith, J., Ryan, J., Hueston, W., 1992. Bovine spongiform encephalopathy: case-control studies of calf feeding practices  
336 and meat and bonemeal inclusion in proprietary concentrates. *Research in Veterinary Science* 52 (3), 325 – 331. doi: [http://dx.doi.org/10.1016/0034-5288\(92\)90032-W](http://dx.doi.org/10.1016/0034-5288(92)90032-W)
- 337
- 338 Wilesmith, J., Wells, G., Cranwell, M., Ryan, J., 1988. Bovine spongiform encephalopathy: epidemiological studies. *Veterinary*  
339 *Record* 123 (25), 638–644. doi: <http://dx.doi.org/10.1136/vr.123.25.638>
- 340 Wilesmith, J. W., Ryan, J. B. M., Arnold, M. E., Stevenson, M. A., Burke, P. J., 2010. Descriptive epidemiological features  
341 of cases of bovine spongiform encephalopathy born after July 31, 1996 in Great Britain. *Veterinary Record* 167 (8), 279–286.  
342 URL <http://veterinaryrecord.bmj.com/content/167/8/279.abstract> doi: <http://dx.doi.org/10.1136/vr.c4552>
- 343 Wilesmith, J. W., Wells, G. A. H., Ryan, J. B. M., Gavier-Widén, D., Simmons, M. M., 1997. A cohort study to examine  
344 maternally-associated risk factors for bovine spongiform encephalopathy. *Veterinary Record* 141 (10), 239–243. doi: <http://dx.doi.org/10.1136/vr.141.10.239>
- 345



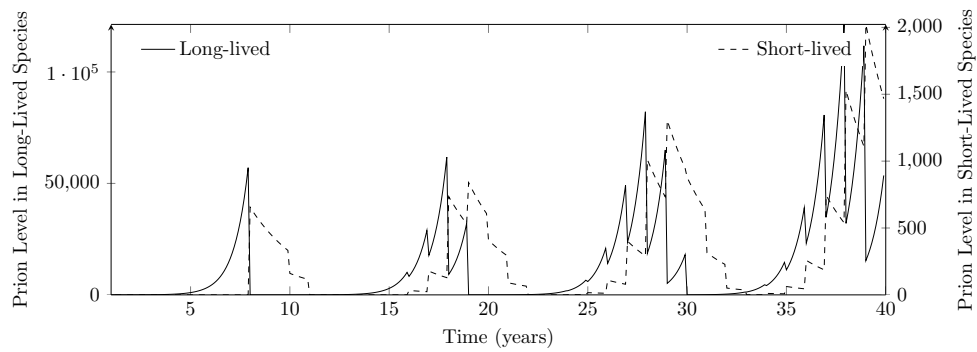
**Figure 1:** Prion growth in a hypothetical animal population with a single-step feedback loop, as in Equation 1. Note that lifespan acts as a threshold on the spread of the disease: for the longer-lived animals (top graph) the disease spreads epidemically because the height of the peaks increases over time, but for animals living just one year less (bottom graph), the peaks decrease until the disease vanishes. Note also the presence of outbreaks and the relation of their period to the lifespan. ( $R = 3$ ,  $c_1 = 0.01$ ,  $c_{i \neq 1} = 0$ ,  $x_{1,t=1} = 1.0$ , 10 time steps/year: smooth growth is assumed within each year with the prion level sampled 10 times per year to produce the figure.)



**Figure 2:** Prion growth in a hypothetical population with a two-species food web, calculated numerically from Equation 5. Growth in the susceptible species is controlled by passing prions to a non-susceptible species in which they decrease; however, under the right conditions this coupling allows the disease to spread through both, maintaining infectivity levels in the non-susceptible species. Equation 7 predicts a lifespan threshold at  $n \approx 7.3$  years. ( $R_s = 0.8$ ,  $R_l = 3$ ,  $b_1 = c_1 = 0.02$ ,  $b_{i \neq 1} = c_{i \neq 1} = 0$ ,  $y_{1,t=1} = 1.0$ ,  $m = 3$ ,  $n = 8$ , 10 time steps/year, where  $m$  and  $n$  are the life-spans of the short- and long-lived species, respectively.)



**Figure 3:** Sensitivity of threshold values. (a) shows the sensitivity of the single-species loop described by Equation 4 to variation in the feedback fraction  $c_1$ , assuming feedback is strictly between the oldest and youngest age-classes. (b) shows the sensitivity of the single-species loop to variation in  $k$ , the initial age at which infected material is fed back, assuming that  $R = 3$ . (c) shows the sensitivity of the two-species loop described by Equation 7 assuming a worst case wherein feed flows from the oldest age-class of the susceptible species to the oldest age-class of the nonsusceptible species and, from there, back to the youngest age-class of the susceptible species. The feedback from the susceptible to the non-susceptible species is set at  $b_m = 0.001$  and the amplification rate of the short-lived species is set at  $R_S = 1$ . The sensitivity graphs of the two-species loop are overlaid on the sensitivity graphs of the single-species loop.



**Figure 4:** Prion growth in a hypothetical population with a two-species food web, calculated numerically from Equation 5. Although one of the species is non-susceptible, the disease spreads through both. Infectious material is fed to multiple age-classes of both species. The cyclic fluctuations and cycle period shown in Figure 2 remain, but become increasingly dispersed and complex as the disease spreads to more and more age-classes. ( $R_s = 0.7$ ,  $R_l = 4$ ,  $b_{i=1,2} = c_{i=1,2,3} = 0.005$ ,  $b_{i \neq 1,2} = c_{i \neq 1,2,3} = 0$ ,  $y_{1,t=1} = 1.0$ ,  $m = 3$ ,  $n = 8$ , 10 time steps/year.)